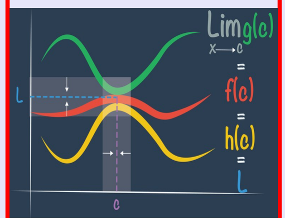
Math 261

Fall 2022

Lecture 26



If $\frac{dx}{dt} = 5$, $\frac{dy}{dt} = 4$, find $\frac{dz}{dt}$ at $(2, 2, 1)$ for

$x^2 + y^2 + z^2 = 9.$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0$$

Divide by 2, Plug in the given information

$$2 \cdot 5 + 2 \cdot 4 + 1 \cdot \frac{dz}{dt} = 0$$

$$\boxed{\frac{dz}{dt} = -18}$$

The height of a triangle increases at 1cm/min

The area increase at $2\ cm^2/min$.

At $\boxed{\text{what rate is its base changing}}$

when height is 10 cm and area is $100cm^2$?

$\frac{dh}{dt}=1$, $\frac{dA}{dt}=2$ $\quad \frac{db}{dt}=?$ $\quad h=10$, $A=100$

Area of Triangle $\quad A=\frac{bh}{2}$ $\quad\quad 2A=bh$

$2(100)=b\cdot 10$

$b=20$

$$\frac{d}{dt}[2A]=\frac{d}{dt}[bh]$$

$$2\cdot\frac{dA}{dt}=\frac{db}{dt}\cdot h+b\cdot\frac{dh}{dt}$$

$$2\cdot 2=\frac{db}{dt}\cdot 10+20\cdot 1$$

$$\frac{db}{dt}\cdot 10=20-4 \quad\quad \frac{db}{dt}=-1.6\ cm/min$$

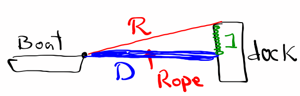
Decreasing

A boat is pulled into a dock by a rope attach to the front of the boat.

The dock is 1 meter higher than the top of the boat.

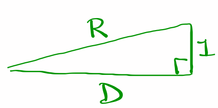
The rope is pulled at the rate of 1m/s,

How fast is the boat approching the dock when it is 8 m from the dock?



$\frac{dR}{dt}=-1\ m/s$

$\frac{dD}{dt}=--$



$$D^2+1^2=R^2$$

$$\cancel{2}D\frac{dD}{dt}+0=\cancel{2}R\frac{dR}{dt}$$

$D=8$

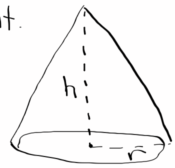
$8^2+1^2=R^2$

$R=\sqrt{65}$

$$8\frac{dD}{dt}=\sqrt{65}(-1)$$

$$\frac{dD}{dt}=\frac{-\sqrt{65}}{8}\ m/s.$$

Gravel is being dumped from a conveyor belt at the rate of $30\ ft^3/min$ and it is making a pile in the form of a Cone with diameter of its base always equa to its height.



$h = 2r \qquad r = \frac{h}{2}$

$\frac{dV}{dt} = 30\ ft^3/min.$

How fast is the height changing when height is 10 ft?

Volume of a Cone

$V = \frac{1}{3}\pi r^2 h$

$3V = \pi r^2 h$

$3V = \pi \left(\frac{h}{2}\right)^2 h$

$12V = \pi h^3$

$12\frac{dV}{dt} = \pi \cdot 3h^2 \cdot \frac{dh}{dt}$

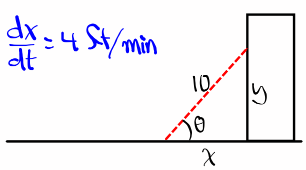
$4 \cdot 30 = \pi \cdot 10^2 \cdot \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{120}{100\pi}$

$\frac{dh}{dt} = \frac{6}{5\pi}\ ft/min.$

A ladder is leaning against a wall. The ladder is 10 ft long. The bottom of the ladder slides away from the wall at the rate of 4 ft/min. How fast is the angle between the ladder and the ground changing when the bottom is 6 ft from the wall?

$\frac{dx}{dt} = 4\ ft/min$



$\frac{d\theta}{dt} = ?$ when $x = 6$

$x^2 + y^2 = 10^2$

$6^2 + y^2 = 10^2 \rightarrow y = 8$

$\cos\theta = \frac{x}{10}$

$10\cos\theta = x$

$-10\sin\theta\,\frac{d\theta}{dt} = \frac{dx}{dt}$

$\sin\theta = \frac{y}{10}$

$\sin\theta = \frac{8}{10}$

$-\cancel{10} \cdot \frac{8}{\cancel{10}} \cdot \frac{d\theta}{dt} = 4$

$\frac{d\theta}{dt} = -\frac{1}{2}$ Rad./min.

Estimate $\sin 31^\circ$

$\sin 31^\circ \approx \sin 30^\circ$

Linear Approximation

$f(x) \approx f(a) + f'(a)\cdot(x-a)$

$f(x) = \sin x$ $\quad f(30^\circ) = \sin 30^\circ = \frac{1}{2}$

$a = 30^\circ$ $\quad f'(x) = \cos x$

$= \frac{\pi}{6}$ $\quad f'(30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$180^\circ = \pi$ Rad

$1^\circ = \frac{\pi}{180}$ Rad

$f(x) \approx \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right)$

$f(31^\circ) \approx \frac{1}{2} + \frac{\sqrt{3}}{2}(31^\circ - 30^\circ)$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}\cdot 1^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\pi}{180}$

$= \frac{1}{2} + \frac{\pi\sqrt{3}}{360}$

$\approx .515$

what about $\sin 31^\circ$

$\approx .515$

Estimate $\sqrt[3]{65}$

$\sqrt[3]{65} \approx \sqrt[3]{64} = 4$

Linear approximation $\quad L(x) = f(a) + f'(a)(x-a)$

$f(x) = \sqrt[3]{x}$ $\quad f(64) = \sqrt[3]{64} = 4$ $\quad = 4 + \frac{1}{48}(x-64)$

$a = 64$ $\quad f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f'(64) = \frac{1}{3\sqrt[3]{64^2}} = \frac{1}{3\cdot 16} = \frac{1}{48}$

$L(x) = 4 + \frac{1}{48}(x-64)$

$\sqrt[3]{65} = 4 + \frac{1}{48}(65-64) = 4 + \frac{1}{48}\cdot 1 \approx 4.021$

use your calc to find

$\sqrt[3]{65} = 4.020725\ldots \approx \boxed{4.021}$

Estimate $\tan 44^\circ$

$\tan 44^\circ \approx \tan 45^\circ \simeq 1$

$L(x) = f(a) + f'(a)(x - a)$

$f(x) = \tan x$ $\quad f(45^\circ) = \tan 45^\circ = 1$

$a = 45^\circ$ $\quad f'(x) = \sec^2 x$

$f'(45^\circ) = \sec^2 45^\circ = (\sqrt{2})^2 = 2$

$\tan 44^\circ \approx 1 + 2(x - 45^\circ)$

$\approx 1 + 2(44^\circ - 45^\circ)$

$= 1 - 2 \cdot 1^\circ$

$= 1 - 2 \cdot \frac{\pi}{180} = 1 - \frac{\pi}{90} = \boxed{.965}$

Using Calc.

$\tan 44^\circ \approx .965688 \text{ - - - -}$ close

$\boxed{.966}$